Progression Towards a Written Method for Addition

Overview:

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. 5 + 3 = 3 + 5
- associative i.e. 5 + 3 + 7 = 5 + (3 + 7)

The fact that it is commutative and associative means that calculations can be rearranged, e.g.
4 + 13 = 17 is the same as 13 + 4 = 17.

To develop an understanding of addition, the progression through learning should be:

Concrete → Pictorial → Abstract

(Using practical objects) (Draw pictures/jottings) (Number line)

Concrete is the 'doing' stage, using concrete objects to solve problems. It brings concepts to life by allowing children to handle physical objects themselves. Every new abstract concept is learned first with a 'concrete' or physical experience.

Pictorial is the 'seeing' stage, using representations of the objects involved in maths problems. This stage encourages children to make a mental connection between the physical object and abstract levels of understanding, by drawing or looking at pictures, circles, diagrams or models which represent the objects in the problem.

Abstract is the 'symbolic' stage, where children are able to use abstract symbols to model and solve maths problems.Once a child has demonstrated that they have a solid understanding of the 'concrete' and 'pictorial' representations of the problem, the teacher can introduce the more 'abstract' concept, such as mathematical symbols.

YR

Early Learning Goal:

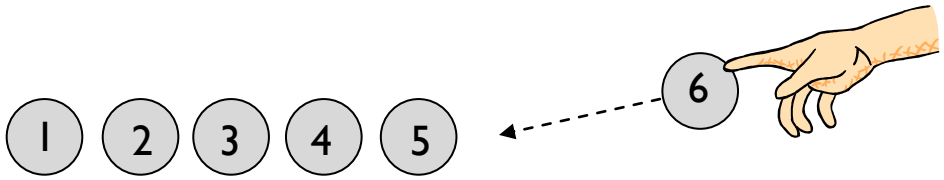
Using quantities and objects, children add two single-digit numbers and count on to find the answer.

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of practical equipment, including small world play, role play, counters, cubes, tens frames, base 10 and Numicon.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating 4 + 2, they are encouraged to count out four counters and count out two counters.

To find how many altogether, touch and drag them into a line one at a time whilst counting.

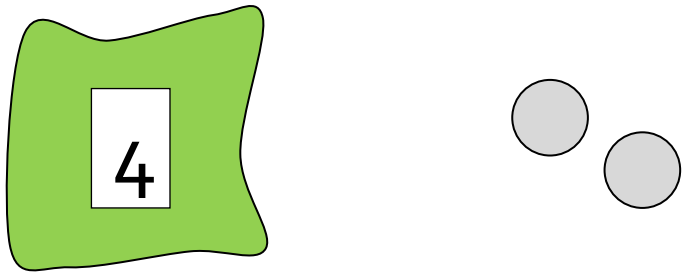


By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating 4 + 2, count out the two groups of counters as before.

Then, cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready may record their own calculations.

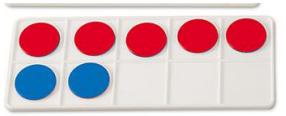
Children will be encouraged to begin to partition numbers in different ways through the Numberland programme. Children will focus on a number a week and begin to understand what that number can be 'made up off' using tens frames and double sided counters/ objects to find different ways that number can be made. e.g. 5 = 3 + 2, 4 + 1 = 5

Reception Addition Examples

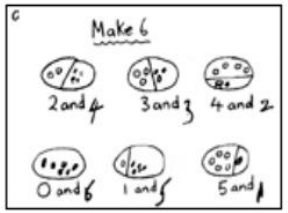
<table>
<tr>
<td>Concrete</td>
<td>
<u>Practical with counters / objects</u>
Counting all method
Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total.

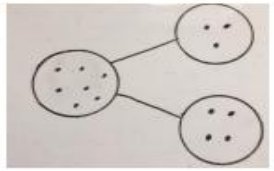
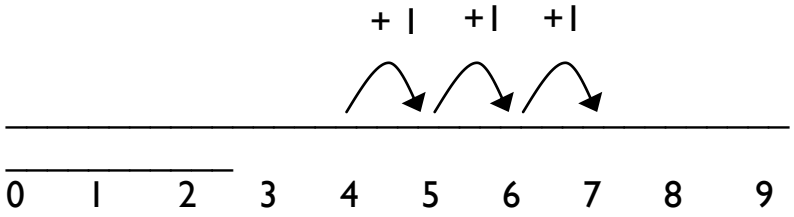
Counting on method
To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted.

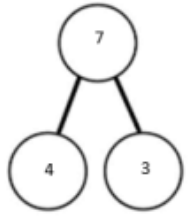

</td>
<td>
<u>Numberland house and garden items totalling 7</u>

</td>
<td>
<u>tens frame</u>

<u>Numicon</u>

</td>
</tr>
<tr>
<td>Pictorial</td>
<td>
<u>Pictures drawn by you</u>

7 = 3 + 4
3 + 4 = 7
</td>
<td>
<u>Pictures drawn by the children</u>


</td>
<td>
<u>Part Part Whole Jottings</u>

7 = 3 + 4
3 + 4 = 7
</td>
</tr>
<tr>
<td>Abstract</td>
<td>
<u>Numberline</u>


</td>
<td>
<u>Number Sentences</u>
4 + 3 = 7
5 + 2 = 7
</td>
<td>
<u>Part Part Whole with numbers</u>


</td>
</tr>
</table>

Y1

End of Year Objective:

Add one-digit and two-digit numbers to 20, including zero (using concrete objects and pictorial representations).

Children will continue to use practical equipment such as; Numicon and tens frames combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to be able to use Base 10 equipment to make teens numbers using separate tens and units.

Reorder numbers in a calculation

In Y1, children need to recognise that they can rearrange an addition, but not a subtraction. They also need to understand that the principle behind reordering a calculation is to make it more efficient, particularly when utilising a counting on strategy. Children need to be encouraged to identify calculations which should be reordered and those that are already in the most efficient format.

Examples of calculations:

8 + 3 doesn't need reordering as the greater number is first already

2 + 7 reorder as 7 + 2

5 + 13 reorder as 13 + 5

11 + 6 doesn't need reordering as the greater number is first already

Equality

Children to develop an understanding of equality using the '=' sign e.g. 6 + □ = 11 11 = 6 + 5 6 + 5 = □ + 4 Shows that what is on the left of the sign is equal in value or amount to what is on the right of the sign.

Concrete

Concrete examples

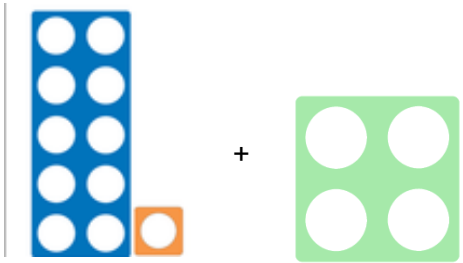
Base 10

When adding 11 and 4, they can make the 11 using a ten rod and a one.

The ones can then be combined to aid with seeing the final total, e.g.

so 11 + 4 = 15. If possible, they should use two different colours of base 10 equipment so that the initial amounts can still be seen.

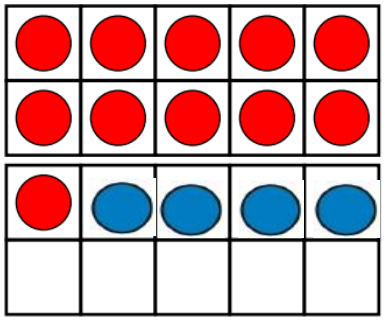
Numicon

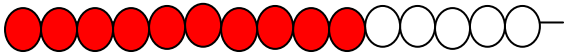


15 = 11 + 4

____ + 4 = 15

Tens frame and Bead String





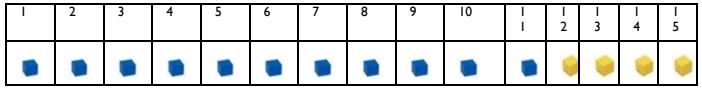
15 = ___ + 11 11 + 4 = 15

____ + 4 = 15

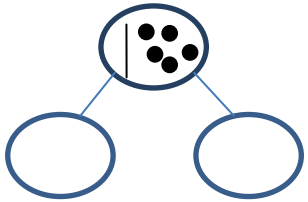
Pictorial

Number Track stage 1

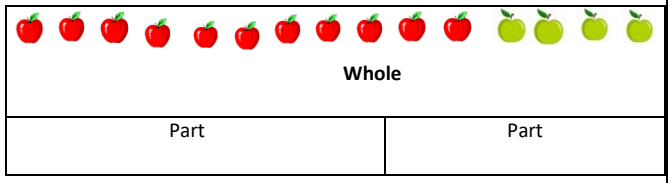
Number Track Stage 2



Part, Part Whole Jottings



Bar Model

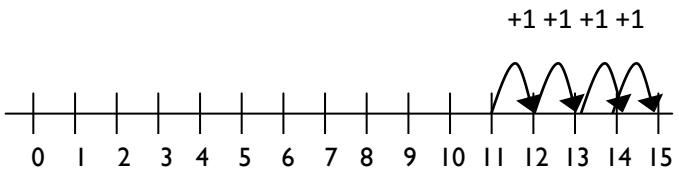


15 = ___ + 11 11 + 4 = 15

____ + 4 = 15

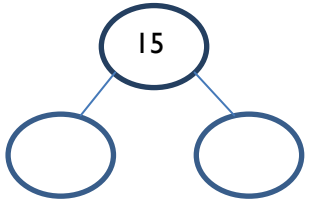
Abstract

Number Line

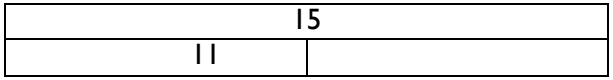


Part Part Whole

15 = ___ + ___



Bar Model



15 = ___ + 11 11 + 4 = 15

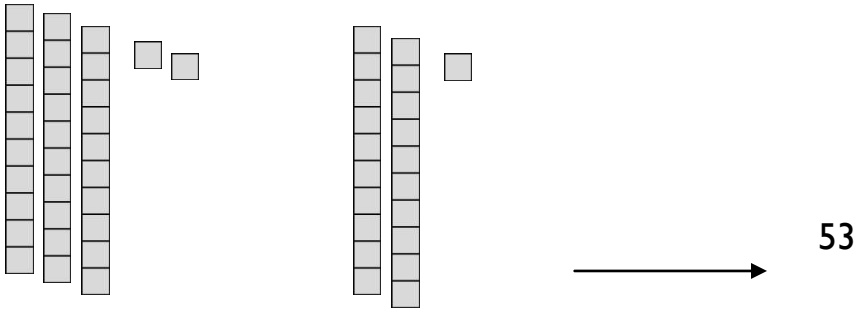
___ + 4 = 15 4 + ___ = 10 + ___

Y2

End of Year Objective:

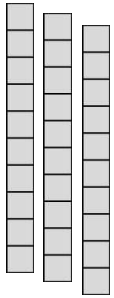
Add numbers using concrete objects, pictorial representations, and mentally, including: a two-digit number and ones; a two-digit number and tens; two two-digit numbers; three one-digit

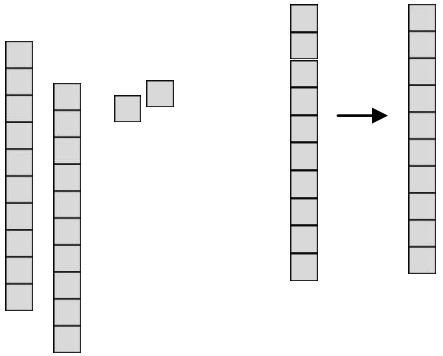
Children will continue to use the Base 10 equipment to support their calculations. For example, to calculate 32 + 21, they can make the individual amounts, counting the tens first and then count on the units.



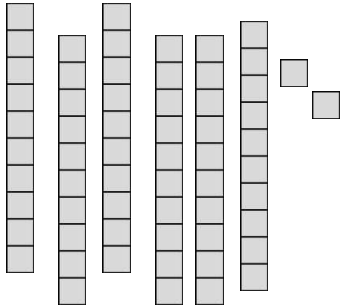
When the units total more than 10, children should be encouraged to exchange 10 units/ones for 1 ten. This is the start of children understanding 'carrying' in vertical addition. For example, when calculating
35 + 27, they can represent the amounts using Base 10 as shown:

Then, identifying the fact that there are enough units/ones to exchange for a ten, they can carry out this exchange:



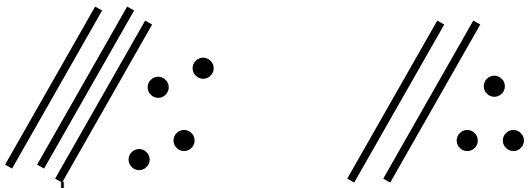


To leave:



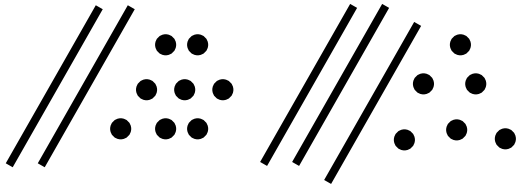
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks).

e.g. 34 + 23 =

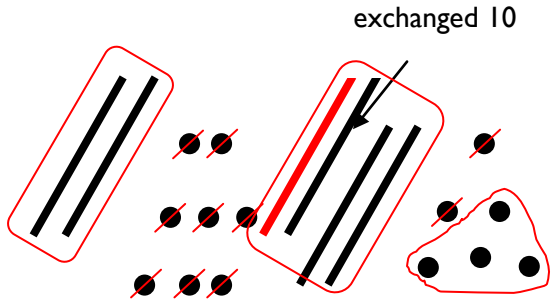


With exchange:

e.g. 28 + 36 =



will become



so 28 + 36 = 64

It is important that children circle the remaining tens and units/ones after exchange to identify the amount remaining.

Year 2 Addition Progression examples (These examples should also be used for exchanging)

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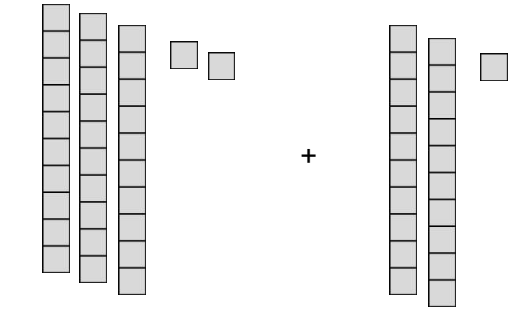
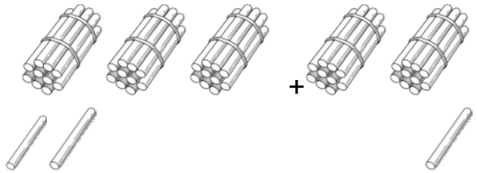
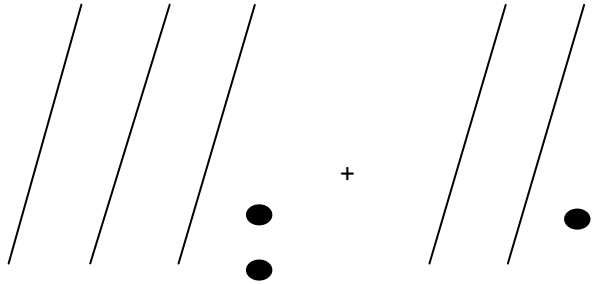
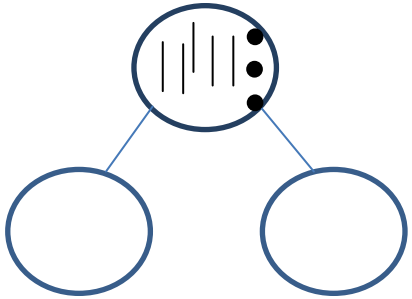
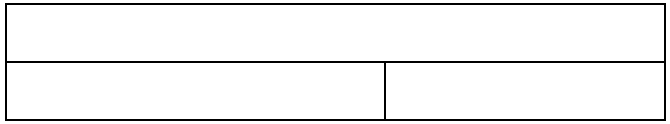
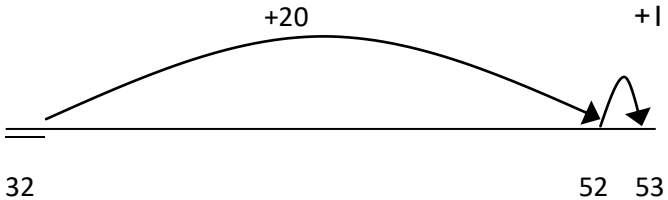
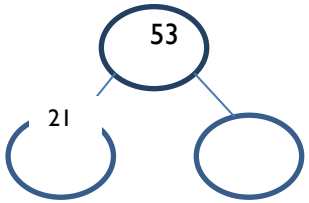
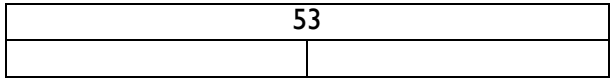
<tr>
<td>Concrete</td>
<td>Base 10
 
32 + 21 = 53
21 + 32 = 53</td>
<td>Straws
 
32 + 21 = 53
32 + ___ = 53</td>
<td>Numicon and Bead String
 
32+ 21 =
21 + 32 =</td>
</tr>
<tr>
<td>Pictorial</td>
<td>Base 10 Jottings
  </td>
<td>Part, Part Whole Jottings
  </td>
<td>Bar Model
  </td>
</tr>
<tr>
<td>Abstract</td>
<td>Number Line
 
32 52 53</td>
<td>Part Part Whole
  </td>
<td>Bar Model
 
53 = ___ + 32 21 + ___ = 53
___ + 40 = 53 13 + ___ = 10 + ___</td>
</tr>
</table>

Y3

End of Year Objective:

Add numbers with up to three digits, using formal written method of columnar addition.*

**Although the objective suggests that children should be using formal written methods, the National Curriculum document states "The programmes of study for mathematics are set out year-by-year for key stages 1 and 2. Schools are, however, only required to teach the relevant programme of study by the end of the key stage. Within each key stage, schools therefore have the flexibility to introduce content earlier or later than set out in the programme of study." p4*

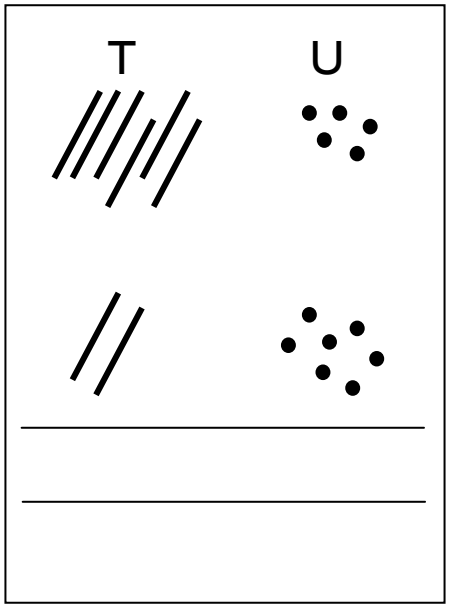
It is more beneficial for children's understanding to go through the expanded methods of calculation as steps of development towards a formal written method.

Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

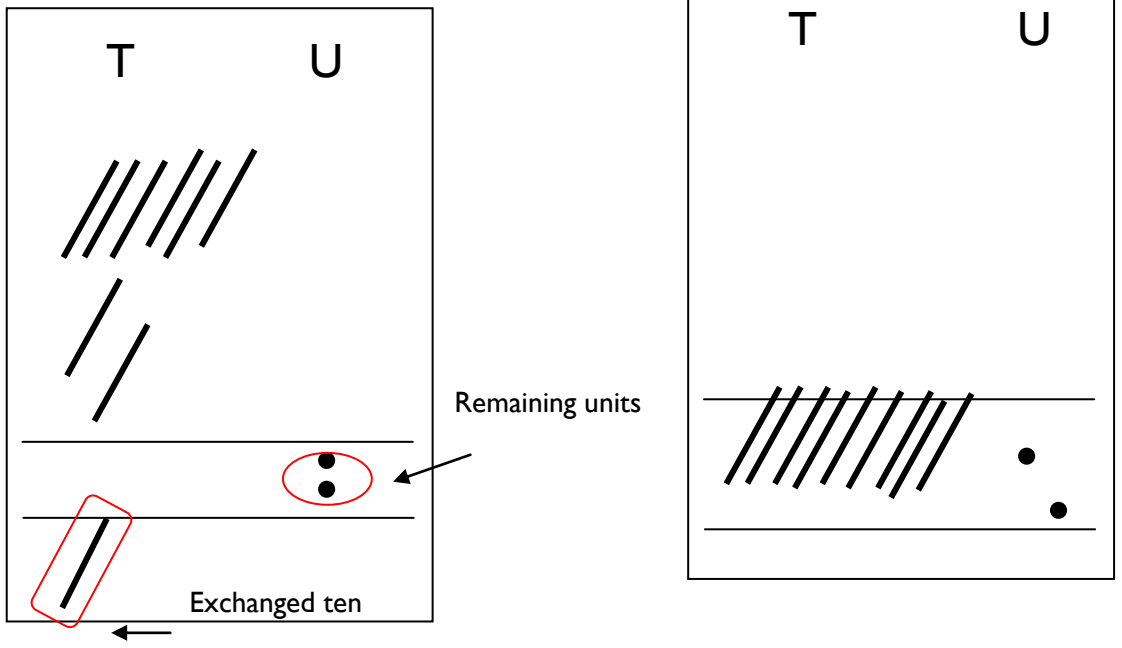
Children should add the **least significant digits** first (i.e. start with the units/ones), and in an identical method to that from year 2, should identify whether there are greater than ten units which can be exchanged for one ten.

They can use a place value grid to begin to set the calculation out vertically and to support their knowledge of exchange between columns (as in Step 1 in the diagram below). e.g. 65 + 27

Step 1



Step 2



Children would exchange ten units/ones for a ten, placing the exchanged ten below the equals sign. Any remaining units/ones that cannot be exchanged for a ten move into the equals sign as they are the units part of the answer (as in the diagram in Step 2 above).

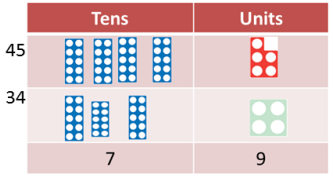
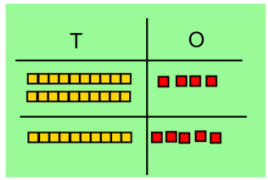
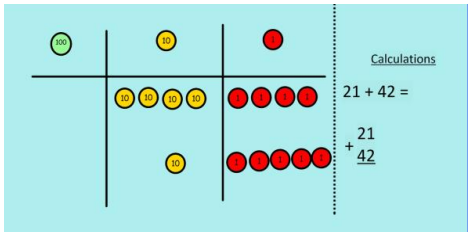
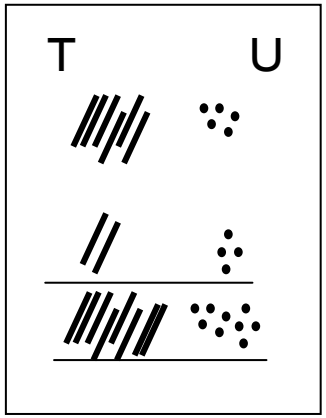
If there are any tens that can be exchanged for a hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer.

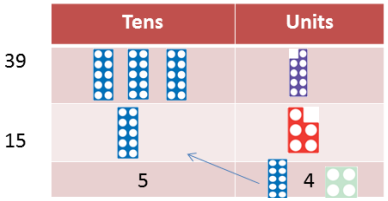
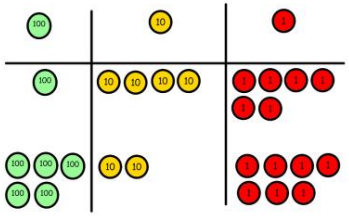
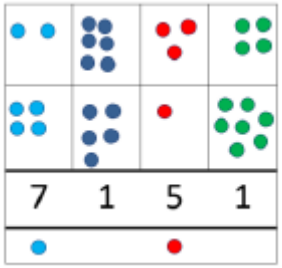
Written method

Step 1	Step 2	Step 3
T U	T U	T U
6 5	6 5	6 5
+ 2 7	+ 2 7	+ 2 7
	2	9 2
	1	1

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside this practical method initially. This should progress to children utilising the written and practical methods alongside each other and finally, and when they are ready, to children utilising just the written method.

By the end of year 3, children should also extend this method for three digit numbers.

Objective/ Strategy	**Concrete**	**Pictoral**	**Abstract**
Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. No Re-grouping	   Model using numicon or base 10. Ensure children are moving their units first before the tens. Move towards using base 10 counters which will help with the extended written method.	Children move to drawing using a tens and one frame. Either with coloured counters or using the sticks and dots to represent the tens and units. 	2 2 3 + 1 1 4 3 3 7

Add and subtract numbers with up to three digits, using formal written methods of columnar addition and subtraction. Re-grouping	  Exchange ten ones for a ten. Model using numicon and place value counters.	 Children can draw a representation of the grid to further support their understanding, carrying the ten underneath the line	2 5 + 1 7 4 2 1 Start by partitioning the numbers before formal column to show the exchange.

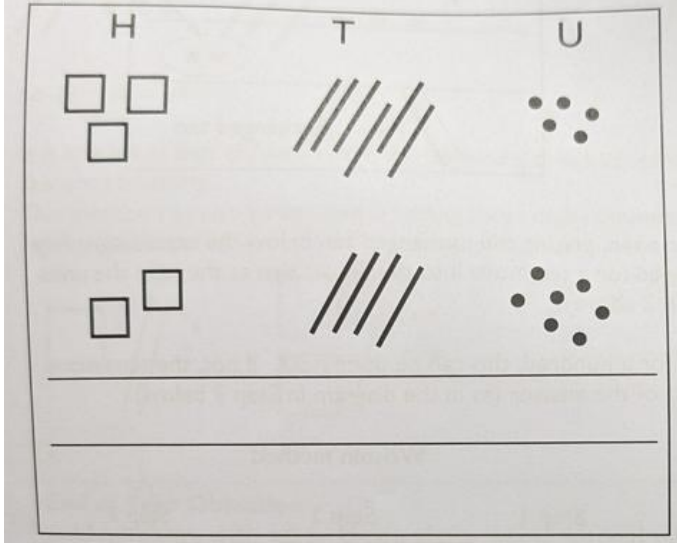
Y4

End of Year Objective:

Add numbers with up to 4 digits *and decimals with one decimal place* using the formal written method of columnar addition where appropriate.

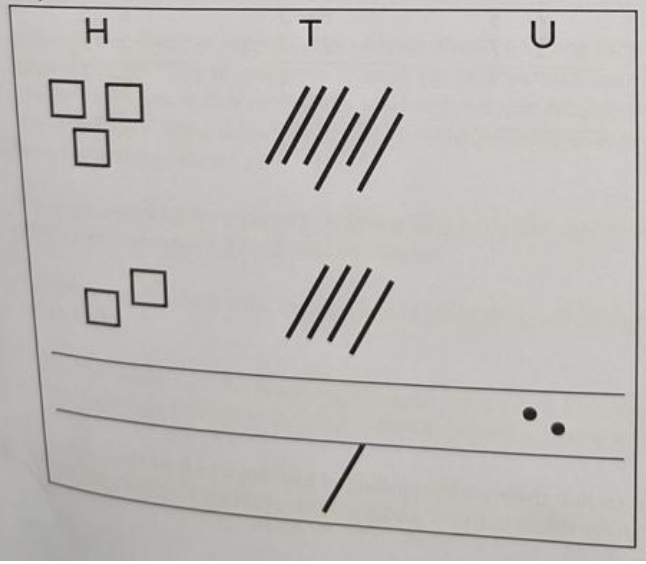
Children will move to year 4 using whichever method they were using as they transitioned from year 3.

Step 1

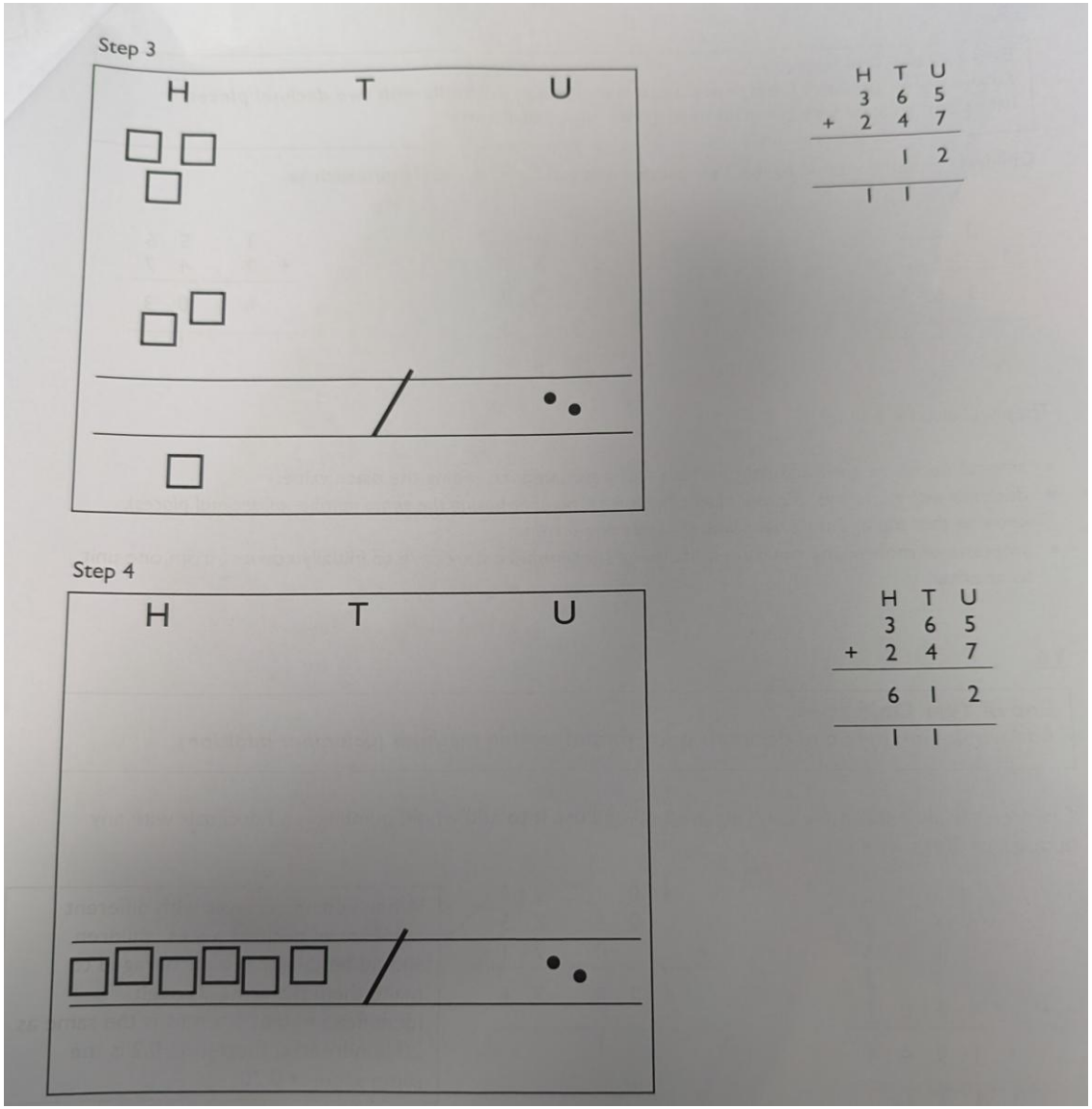


	H	T	U
	3	6	5
+	2	4	7

Step 2



	H	T	U
	3	6	5
+	2	4	7
			2
		1	

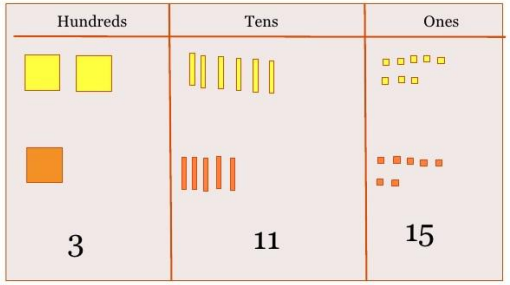
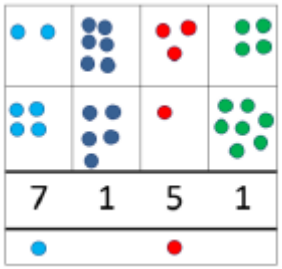
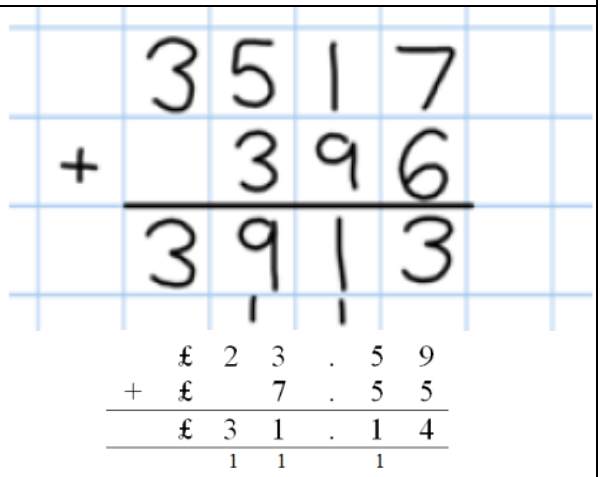


	H	T	U
	3	6	5
+	2	4	7
		1	2
	1	1	

	H	T	U
	3	6	5
+	2	4	7
	6	1	2
	1	1	

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another.*

Objective/ Strategy	**Concrete**	**Pictoral**	**Abstract**
add numbers with up to 4 digits	Children continue to use base 10 or place value counters to add, exchanging ten ones for a ten and ten tens for a hundred and ten hundreds for a thousand. 	 Draw representations using the place value grid.	 As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

Y5

End of Year Objective:

Add whole numbers with more than 4 digits *and decimals with two decimal places*, including formal written methods (columnar addition).

Children should continue to use the carrying method to solve calculations such as:

```
  3364
+  247
------
  3611
------
   11
```

```
  3121
    37
+  148
------
  3306
------
   11
```

```
  3.56
+ 2.47
------
  6.03
------
   1
```

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Y6

End of Year Objective:

Add whole numbers and decimals using formal written methods (columnar addition).

Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

```
    4 2
6 4 3 2
    7 8 6
        3
+   4 6 8 1
```

```
  4 0 1 . 2 0
    2 6 . 8 5
+     0 . 7 1
-------------
  4 2 8 . 7 6
    1
```

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to three decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.